

Year 12 Homework 1



Algebraic Fluency 1 | Difference of Two Squares

Factorise the following expressions:

1. $x^2 - 1$

2. $q^2 - 25$

3. $25a^2 - 36$

4. $x^2 - y^2$

5. $36x^2 - 49a^2$

6. $(x + 3)^2 - 1$

7. $(2x + 1)^2 - (2x - 1)^2$

8. $x^4 - 9$

9. $p^2r^2 - q^2s^2$

10. $x^2 + 9$

11. $9 - 4b^2$

12. $8x^2 - 50$

13. $27x^2 - 300$

14. $2x^4 - 2y^4$

(one of these doesn't factorise!)

Algebraic Fluency 2 | Simplify the Algebraic Fractions

Factorise (if necessary) the numerator and denominator to cancel down the fractions:

1. $\frac{6x^2-6}{3x+9}$

2. $\frac{10x+20}{10x+14}$

3. $\frac{5x+15}{20x+60}$

4. $\frac{x^2y^3z^5}{x^5yz^3}$

5. $\frac{2(x+2)(x+4)}{3(x+4)(x-3)}$

6. $\frac{6x^2-6}{3x+3}$

7. $\frac{x^2+3x-10}{x^2+6x-16}$

8. $\frac{x^2-x-6}{x^2+5x+6}$

9. $\frac{x^2+x-6}{x^2+6x+5}$

10. $\frac{4x^2-1}{2x^2-5x-3}$

(one of these doesn't cancel down!)

1. Set out your working using the titles in bold:

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

Specialise

Shade in 4 blocks in the following pattern:

	28	
33		35
	40	

Multiply the top and bottom and the left and right.

Find the difference between the pairs.

Try another set with the same pattern.

Conjecture

What pattern do you notice? Explain. Try it for a different set

Generalise

Can you prove your conjecture algebraically?

- Pick another pattern of squares from the grid above and **Specialise**, **Conjecture** and **Generalise**. Be creative!
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1	2	3	4	5	6	...
1+a	2+a	3+a	4+a	5+a	6+a	...
1+2a	2+2a	3+2a	4+2a	5+2a	6+2a	...
1+3a	2+3a	3+3a	4+3a	5+3a	6+3a	...
1+4a	2+4a	3+4a	4+4a	5+4a	6+4a	...
...

Now the grid is generalised to have length a.

Specialise

Shade 4 blocks in the same pattern as in question 1.

Multiply the top and bottom and the left and right and find the difference between them.

Do it for another set of 4 blocks in the same pattern. Now **Conjecture** a result.

Generalise* by proving this result algebraically.

Direct Proofs

4. Prove that the sum of any two even numbers is always even
5. Prove that the product of any two odd numbers is always odd
6. Prove that the sum of 5 consecutive numbers is always a multiple of 5.
7. Prove that the sum of 7 consecutive numbers is always a multiple of 7
8. *Prove that the sum of any odd number of consecutive numbers is always a multiple of that odd number.
9. Prove that the product of two consecutive odd numbers is always one less than a square number.
10. Prove that the difference between consecutive odd square numbers is never prime.
11. Prove that the difference between square numbers can only be prime if they are consecutive square numbers.
12. Prove that the only prime number that you can get by taking 1 away from a square number is 3.
13. Prove that the sum of two rational numbers is always rational

Counter Examples

14. All prime numbers are odd
15. $x^2 < 9 \Rightarrow x < 3$
16. The difference between consecutive square numbers is always prime
17. The sum of any even number of consecutive numbers is always a multiple of that even number.
18. If you add 1 to an even square number then you get a prime number.
19. $\{n: n \in \mathbb{N}\}$
For any n , either $6n - 1$ or $6n + 1$ is prime
20. If you add even square numbers to 163 then you get a prime number

Research

For the following questions use the internet or books to find a proof of the following. Don't just copy it – try to make sense of the steps so that you could also explain it.

21. Find a proof of Pythagoras' theorem.

22. Prove the sine rule for the area of a triangle ($A = \frac{1}{2}ab \sin C$)

23. Find a counter example for the following statement:

An irrational number to the power of an irrational number is always an irrational number.

24. Find a geometric proof of the difference of two squares

25. Prove the quadratic formula:

For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*This symbol means the question is very tricky. Please see how far you get but don't let these questions hold you back from moving on.